Analysis of Force Vectors in a Vertical Circle.

There are numerous examples of vertical circular motion. Some obvious ones include:

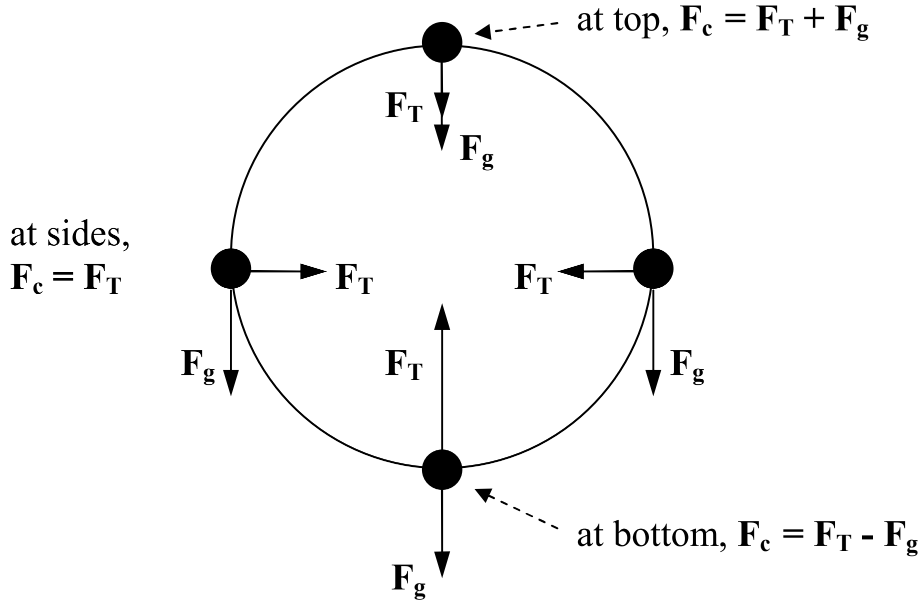
- roller coasters and toy cars doing loop-the-loops;
- twirling a mass attached to a cord, much like a 360° pendulum;
- riding a Ferris wheel;
- driving a vehicle over a round hill.

For these and all vertical circular motion problems, the net force is not always centripetal, and is therefore not dealt with in Physics 12. For our purposes we will only consider those forces which affect the *net centripetal force*. This means we can only analyze forces acting on a mass at specific positions in the vertical circle.

Objects that travel in vertical, circular paths typically have a minimum of two forces acting on them. By examining free-body diagrams of these forces, the net centripetal force can be determined mathematically.

A. Bob attached to a string.

Examine the forces on a bob going in a vertical circle; note that forces pointing **into** the circle center are *positive*, while forces pointing **away** from the center are *negative*. As well, any *tangential* force has no effect on the net centripetal force.



The tension in the string $\mathbf{F_T}$ that connects the bob to the center must supply the force needed to provide the net centripetal force and to support the bob's weight.

At the <u>bottom</u>, the tension in this string is $\mathbf{F_T = F_c + F_g}$

At the <u>side</u>, $\mathbf{F_g}$ is tangential, so $\mathbf{F_T = F_c}$

At the <u>top</u>, the weight contributes towards $\mathbf{F_c}$: $\mathbf{F_T = F_c - F_g}$

Example #4: A 0.90 kg mass attached to a cord is whirled in a vertical circle of radius 2.5 m.

a) **Find the tension in the cord at the top of the circle if the speed of the mass is 8.7 m/s.**
b) **Find the tension in the cord at the bottom of the circle if the speed is maintained at 8.7 m/s.**

(see Circular Motion Ex 4 for answer)

Note: at the top of these loops, there is a certain minimum speed, sometimes called the ***critical velocity***, that will <u>just</u> keep the mass going around the loop on the track. It is the speed at which the track exerts no normal/tension force, so that the needed centripetal force equals the weight of the mass, or $\mathbf{F_c = F_g}$. If the mass goes slower, then $\mathbf{F_c < F_g}$ and the mass will *fall*.

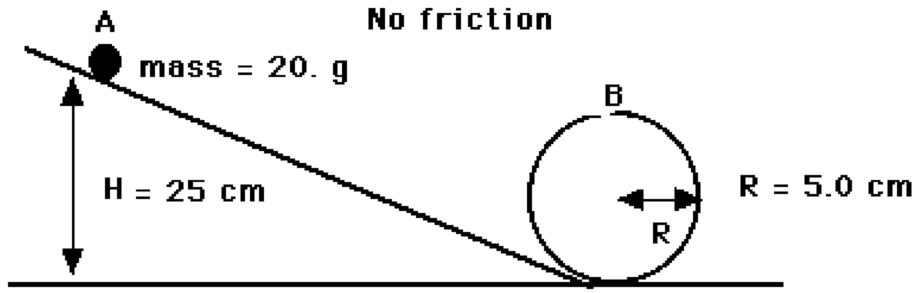
Example #5: The same system above is now whirled at a slower rate.

a) **What minimum speed must it have at the top of the circle so as not to fall from the circular path?**
b) **At the speed in (a) and neglecting any friction, how fast will the object be going at the bottom of the circle?**
c) **What is the tension in the cord at the bottom at this speed?**

(see Circular Motion Ex 5 for answer)

B. A vertical loop of track (e.g., on a roller coaster).
A vehicle going around a track loop will have the normal force of the track ($\mathbf{F_N}$) acting on the mass and contributing to the centripetal force in the same way as $\mathbf{F_T}$ does. Simply substitute $\mathbf{F_N}$ in for $\mathbf{F_T}$ in the above equations.

Example #6: A 20. gram steel ball-bearing on a rail rolls from rest at point A, as shown below.



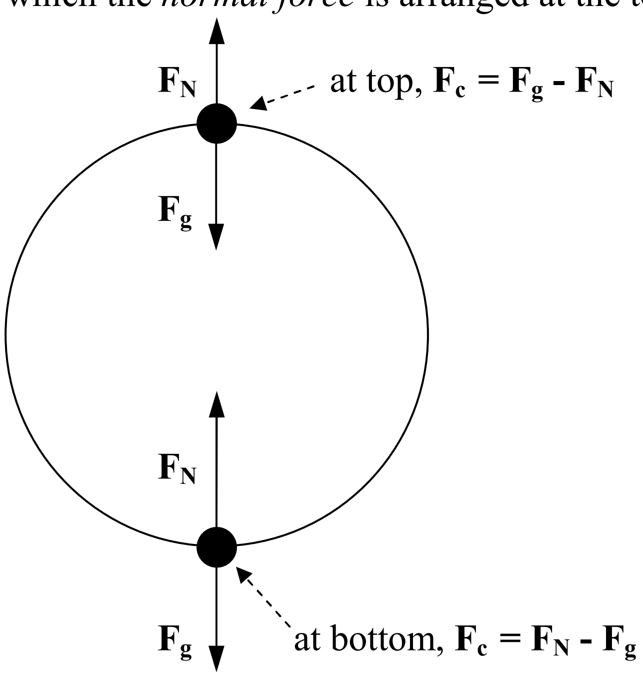
Assuming negligible friction, if h = 0.25 m and R = 0.050 m,

a) what is the speed of the bearing at point B?

b) what normal force must the rail exert on the bearing at B?

(see Circular Motion Ex 6 for answer)

C. The ferris wheel.
In this situation, the forces are arranged slightly differently; note in particular the way in which the *normal force* is arranged at the top:



At these two locations, the normal force acts up to counteract gravity, but each equation is different (remember, pointing *in* is positive).

As well, at the top there is a *maximum* critical speed where the person will just remain in her seat, without flying off tangentially. The normal force is 0, so once again, at this speed: $\mathbf{F_c = F_g}$

Note that this analysis also works for vehicles driving up and over a hill.

Example #7: A 62 kg student drives his 450 kg car at 25 m/s up towards the top of a hill of radius 70. m.
a) What normal force will the driver's seat exert on him at the top of the hill?
b) How fast can he drive his car over the hill without being airborne?

(see Circular Motion Ex 7 for answer)